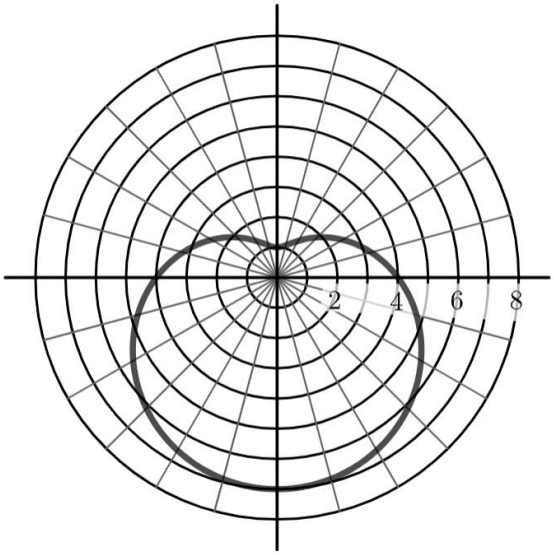
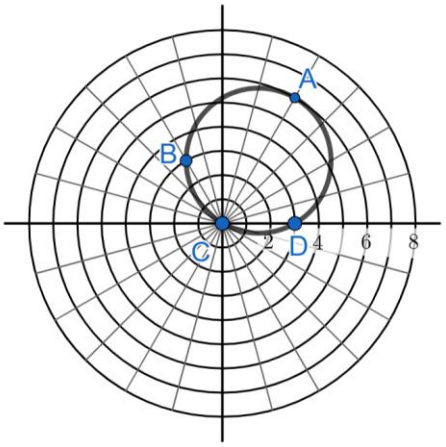
More Polar Graphing + FRQ3 Review Homework (3.14+3.15)

1. The figure to the right shows the graph of the polar function $r = f(\theta)$, for $0 \le \theta \le 2\pi$, in the polar coordinate system. Which of the following could be an expression for $f(\theta)$?
   (A) $3 - 4\sin(\theta)$
   (B) $3 + 4\sin(\theta)$
   (C) $4 - 3\sin(\theta)$
   (D) $4 + 3\sin(\theta)$



2. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = -8\sin(2\theta)$, in the polar coordinate system for $0 \le \theta \le 2\pi$. Which of the following statements is true about the distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin?
   (A) The distance is increasing for $\pi \le \theta \le \frac{5\pi}{4}$, because $f(\theta)$ is negative and decreasing on the interval
   (B) The distance is increasing for $\frac{5\pi}{4} \le \theta \le \frac{7\pi}{4}$, because $f(\theta)$ is positive and increasing on the interval
   (C) The distance is decreasing for $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$, because $f(\theta)$ is positive and decreasing on the interval
   (D) The distance is decreasing for $0 \le \theta \le \frac{\pi}{4}$, because $f(\theta)$ is negative and decreasing on the interval

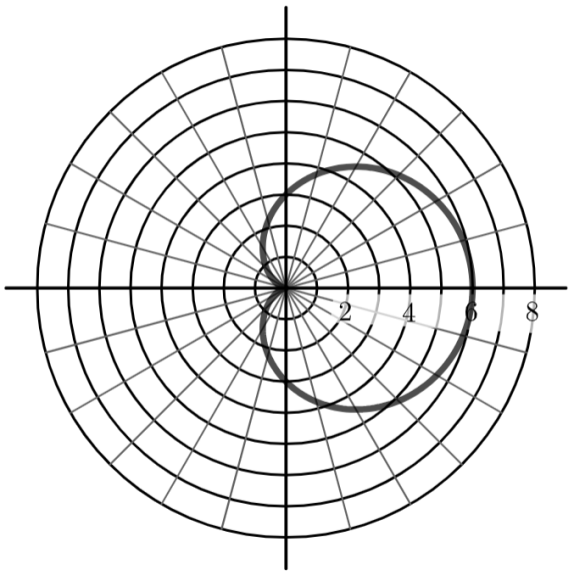
3. The figure shows the graph of the polar function $r = f(\theta)$, where $f(\theta) = 6\cos\left(\theta - \frac{\pi}{3}\right)$, in the polar coordinate system for $0 \le \theta \le 2\pi$. There are four points labelled A, B, C, and D. If the domain of f were restricted to $\pi \le \theta \le \frac{5\pi}{3}$, which of the following describes the portion of the given graph that remains?
   (A) The portion of the graph from B to D that includes point C
   (B) The portion of the graph from D to B that includes point A
   (C) The portion of the graph from C to B that includes points D and A
   (D) The portion of the graph from B to C that lies only in Quadrant II



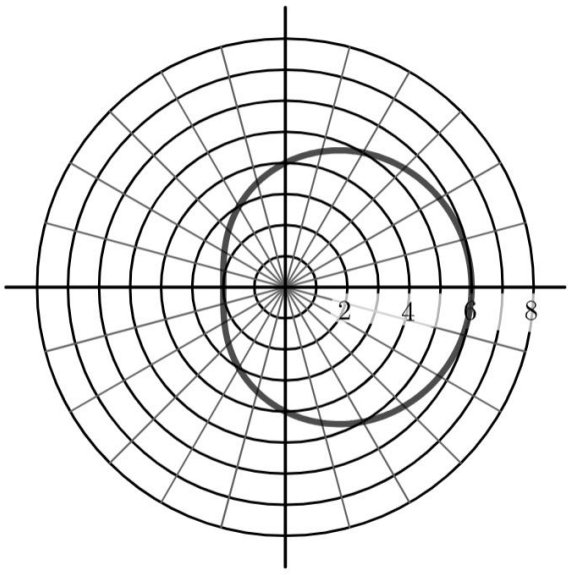
4. The polar function $r = f(\theta)$, where $f(\theta) = 1 - \sin(\theta)$ is graphed in the polar coordinate system for $0 \le \theta \le 2\pi$. Give all intervals on which the distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin is decreasing.

5. Which of the following is the graph of the polar function $r = f(\theta)$ where $f(\theta) = 2 + 4\cos\theta$, in the polar coordinate system for $0 \leq \theta \leq 2\pi$?

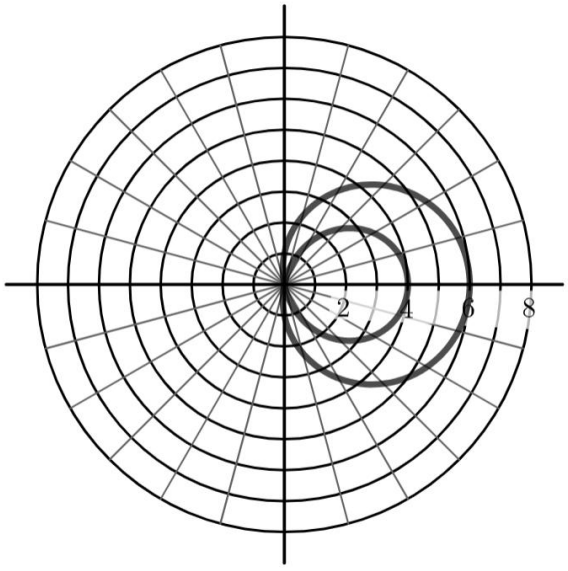
(A)



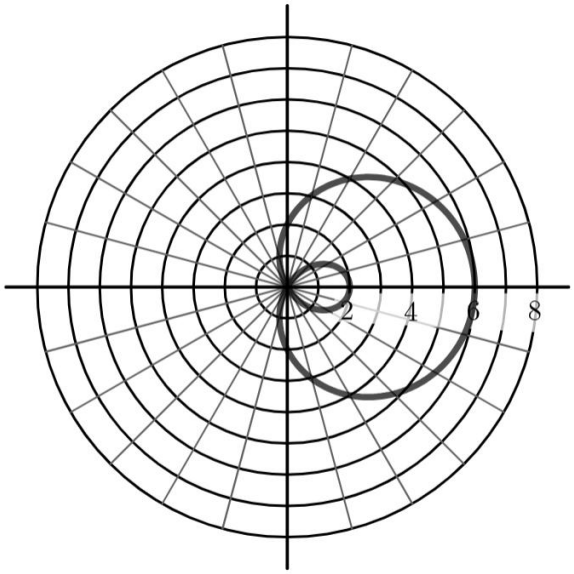
(B)



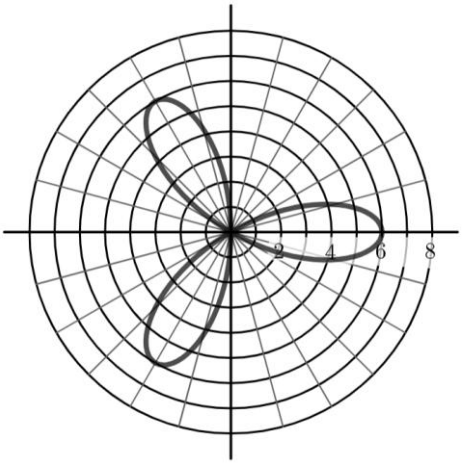
(C)



(D)



6. The figure shows the graph of the polar function $r = f(\theta)$, where $f(\theta) = 6\cos(3\theta)$, in the polar coordinate system for $0 \leq \theta \leq 2\pi$. If the domain of f were restricted to $\frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3}$, the portion of the graph that remains would lie in two quadrants. Which two quadrants would this be?

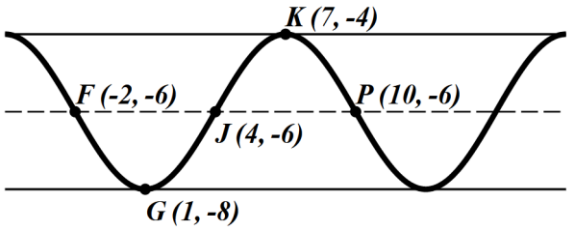


Questions #7 and #8 each give a function $f(\theta)$ and an interval. Determine if over the given interval the distance between the point $\big(\theta, f(\theta)\big)$ and the origin is increasing or decreasing.

7. $f(\theta) = -1 - 2\sin(\theta)$ over the interval $\frac{\pi}{2} \le \theta \le \frac{7\pi}{6}$

8. $f(\theta) = 1 + 2\sin\left(2\theta + \frac{\pi}{6}\right)$ over the interval $\frac{\pi}{2} \le \theta \le \frac{2\pi}{3}$

9. The graph of the sinusoidal function h is shown below.



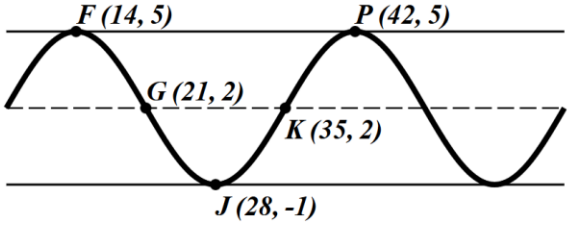
(a) This function can be written in the form $h(t) = a\sin\big(b(t+c)\big) + d$. Find a suitable value of a, b, c, and d.

(b) The t-coordinate of K is t_1 and the t-coordinate of P is t_2. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing
b. h is positive and decreasing
c. h is negative and increasing
d. h is negative and decreasing

(c) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

10. The graph of the sinusoidal function h is shown below.



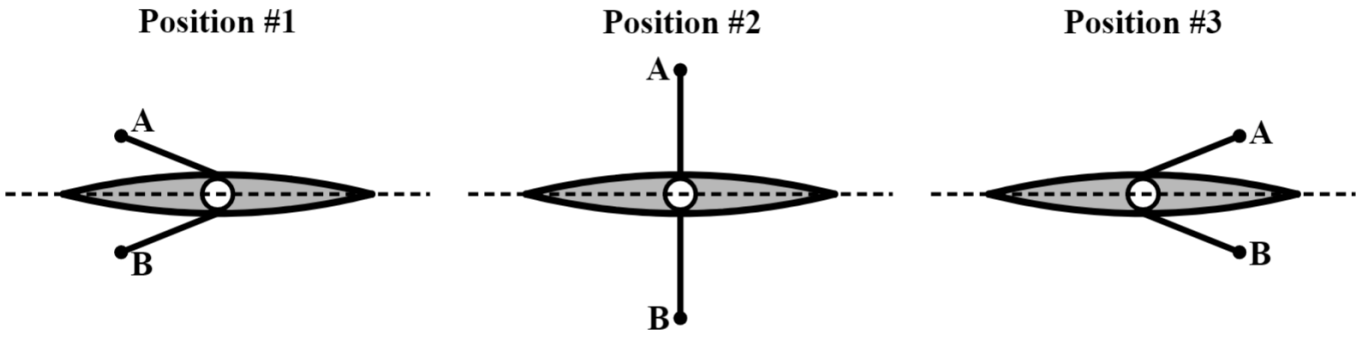
(a) This function can be written in the form $h(t) = a\cos\big(b(t+c)\big) + d$. Find a suitable value of a, b, c, and d.

(b) The t-coordinate of K is t_1 and the t-coordinate of P is t_2. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing
b. h is positive and decreasing
c. h is negative and increasing
d. h is negative and decreasing

(c) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

11. Bryan Passwater's FRQ 3 Task Model D:



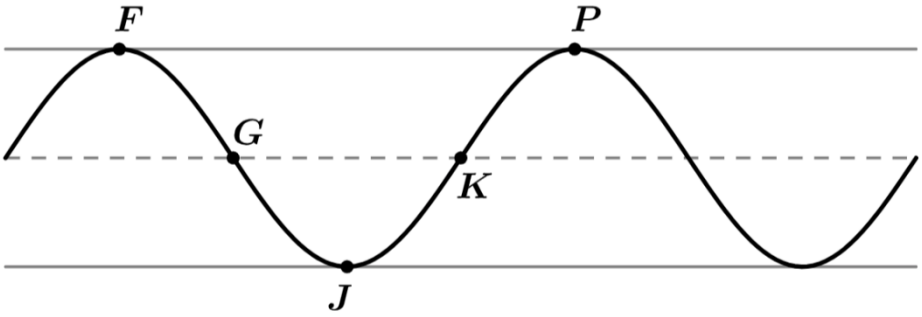
Note: Figure is not drawn to scale

The figure shows an overhead view of a canoe with its paddles extended out on both sides of the canoe. Point A and point B are at the end of each paddle. As the occupant in the canoe rows, the position of the two paddles moves from position #1 to position #3, before returning back to position #1. At both position #1 and position #3, the distance between A and B is 5 feet. The greatest distance between A and B occurs at position #2, where A and B are each 6 feet from the dashed centerline of the canoe.

At time $t = 0$ seconds, the paddles are in position #1. The paddles then rotate through position #2 to position #3. At time $t = 3$ seconds, the paddles reach position #3 for the first time. Then, the paddles rotate and reach position #1 again at time $t = 6$ seconds. As the occupant paddles, the distance between A and B periodically increases and decreases.

The sinusoidal function h models the distance between A and B, in feet, as a function of time t, in seconds.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points $F, G, J, K,$ and P are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $\big(t, h(t)\big)$ for the five points F, G, J, K, and P.



(B) The function h can be written in the form $h(t) = a\sin\big(b(t + c)\big) + d$. Find the values of the constants a, b, c, and d.

(C) Refer to the graph of h in part (A). The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h?
   a. h is positive and increasing.
   b. h is positive and decreasing.
   c. h is negative and increasing.
   d. h is negative and decreasing.

(ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

More Polar Graphing + FRQ3 Review HW Answers

1. C
2. A
3. B
4. $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$
5. D
6. Quadrant II and Quadrant III
7. Decreasing as $f(\theta)$ is negative and increasing
8. Increasing as $f(\theta)$ is negative and decreasing.
9. (a) $a = 2 \quad b = \frac{\pi}{6} \quad c = -4 \quad d = -6$ (Note: these are not the only correct values, but perhaps the most straightforward choices)
   (b) d. h is negative and decreasing
   (c) h is concave down on the interval (t_1, t_2), so the rate of change of h is decreasing on this interval.
10. (a) $a = 3 \quad b = \frac{\pi}{14} \quad c = -14 \quad d = 2$ (Note: these are not the only correct values, but perhaps the most straightforward choices)
    (b) a. h is positive and increasing
    (c) h is concave down on the interval (t_1, t_2), so the rate of change of h is decreasing on this interval.
11. (A) $F: (1.5, 12)$
    $G: (2.25, 8.5)$
    $J: (3, 5)$
    $K: (3.75, 8.5)$
    $P: (4.5, 12)$
    (B) $a = 3.5 \quad b = \frac{2\pi}{3} \quad c = -3.75 \quad d = 8.5$
    (C) (i) b. h is positive and decreasing
    (ii) The graph of h is concave up on the interval (t_1, t_2). Thus, the rate of change of h is increasing on the interval (t_1, t_2).